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# OPTIMUM DESIGN OF PRESSURE STABILIZED BEAMS

by

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The minimum weight design problem for pressure-stabilized beams is formulated and solved in order to provide the designer some guidance in the use of the design analysis capability developed and reported previously. The weight is minimized subject to four inequality constraints to give the inflation pressure, cross-section radius and fabric density corresponding to the minimum weight. It is shown that high pressures, small radii, and low fabric density give minimum weight. In addition, it is found that high fabric strength and low fabric stiffness per unit weight should be used for minimizing the weight.

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## PREFACE

This work is part of an ongoing investigation of the behavior of pressurized structural elements under load with the ultimate objective of making possible the use of such elements in the support structure of Army tents. The work was funded under the In-House Laboratory Independent Research program as a work unit entitled, "Study of the Stability of Pressure-Stabilized Arches and their Structural Assemblies." In the reference citations the organizations "US Army Natick Laboratories" and "US Army Natick Development Center" refer to the organization now called the "US Army Natick Research and Development Command."

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# OPTIMUM DESIGN OF PRESSURE STABILIZED BEAMS

## INTRODUCTION

Over the past several years the pressurized rib concept for tent support structures has been under development for use in Army field tentage. The pressurized rib concept which is illustrated in Figure 1 consists of a frame of pressure-stabilized beams and arches covered by a lightweight fabric environmental barrier. This concept differs from the more familiar double-wall air-supported tents in that there are fewer pressure-stabilized structural elements, and those used have a smaller cross-section, thus reducing the weight of the support structure. This reduction in size and number of structural elements is made possible by increasing the inflation pressure level. In present double-wall shelters, pressures on the order of 5 kPa are used, while it is anticipated that pressures in the range 200 to 500 kPa will be used with the pressurized rib concept. In addition to the weight reduction possible, it is also believed that the concept will achieve reductions in the packaged bulk because the structural elements are made from flexible materials such as fabrics so they can be compactly folded when not inflated. While having these advantages over present tentage it retains the rapid erection and striking characteristics of all air-supported structures. It is also anticipated that the pressurized rib concept can be fabricated with sufficient air retention capability to eliminate the need for the dedicated air supply required of current air-supported tents. Several years ago a systems analysis of the Army's shelter requirements found that the pressurized rib concept showed great promise for meeting the Army's tentage needs. It was this result that initiated the study of the concept. A recent study, reference 1, evaluated a number of tent support structure concepts with regard to mobility, habitability and cost, and found the pressurized rib concept as one of the most promising, thus establishing its logistical feasibility. The technical feasibility of the concept was established by the results presented in reference 2. This study clearly established the possibility of making inherently stable frame assemblies using pressure-stabilized arches and beams and demonstrated that such frames could withstand the Army operational snow load within the pressure range anticipated for use with the concept. The major emphasis of the work completed to date was the development of a design capability for pressure stabilized beams and arches and for tent structures using

[1] Arthur Johnson; Comparative Evaluation of Concepts for Modular Tentage; US Army Natick Research & Development Command, Technical Report NATICK/TR–78/009, 1978 (AD A055347)

[2] Earl C. Steeves; Fabrication and Testing of Pressurized Rib Tents; US Army Natick Research & Development Command, Technical Report NATICK/TR-79/008

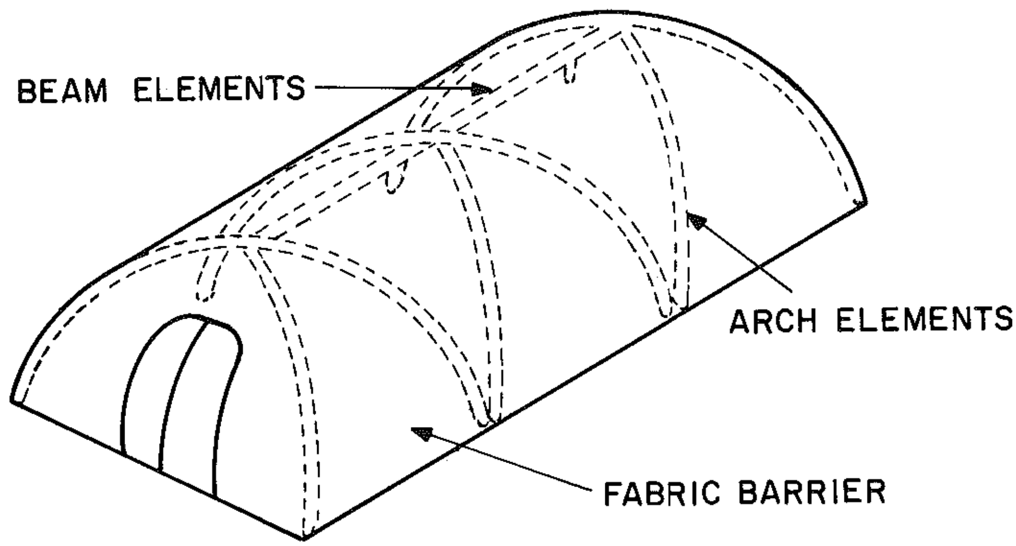



Figure 1. Tent Concept Using Pressure Stabilized Structural Elements

the pressurized rib concept. The work on the beams and arches is presented in references 3, 4, and 5 and provides a theory for the prediction of the deformation and load-carrying capability of these structural elements as a function of the element size, pressure level, and material properties. Also included in these reports are experimental results that confirm the accuracy of the theoretical predictions. To provide a design capability for complete tents using the pressurized rib concept, a finite element for pressure stabilized beams was developed and reported in reference 6. The finite element was adapted to a finite element computer code for the analysis of frame-supported tents.

We thus have adequate analysis capability, but as this capability was used, it became apparent that the tent designer needed some help with synthesis type of questions: Should higher pressure level and smaller cross-section sizes be used, do high or low stiffness material give the better structure? To give an answer to these questions, an optimum design study was undertaken using structure weight as the objective function to be minimized or optimized. The objective of this report is to describe this study of the minimum weight design of a pressure-stabilized beam under a uniform load. The beam weight is minimized as a function of inflation pressure level, cross-section radius, and fabric weight density. These results are used to infer the influence of fabric stiffness on weight minimization.

## ANALYSIS

### Problem Formulation

The problem is stated in two parts; the objective function and the constraints, with the beam weight as the objective function. Both the objective function and the constraints are expressed in terms of the independent variables, pressure, cross-section radius, and the material area density (p, a, d). The structural element considered is a uniformly

[3] Earl C. Steeves; A Linear Analysis of the Deformation of Pressure-Stabilized Beams; US Army Natick Laboratories, Technical Report 75-47-AMEL, 1975 (AD A006493)

[4] Earl C Steeves; Behavior of Pressure Stabilized Beams Under Load, US Army Natick Development Center, Technical Report 75-82-AMEL, 1975 (AD A010702)

[5] Earl C. Steeves; The Structural Behavior of Pressure-Stabilized Arches, US Army Natick Research & Development Command, Technical Report NATICK/TR-78/018; 1978 (AD A063263)

[6] Earl C. Steeves; Pressure Stabilized Beam Finite Element, US Army Natick Research & Development Command, Technical Report NATICK/TR-79/002; 1978 (AD A064732)

loaded, simply supported beam of length 2L, and using the notation shown in Figure 2, the weight, W, of the beam is given as

$$W = 4\pi a L d \tag{1}$$

This is the function to be minimized, subject to the constraints that the wrinkling load failure criterion be satisfied and that the fabric not rupture. We now proceed to express these constraints mathematically in terms of the independent variables, pressure, radius and fabric density (p, a, d). The wrinkling load failure criterion which is discussed in reference 3 assures that the design is chosen so that the applied load can be supported and requires that the absolute value of the compressive bending stress, Nx, be less than the axial stress due to pressurization. This is expressed mathematically as

$$pa \geqslant 2N_X \tag{2}$$

We find from reference 3 that for the case of a simply supported beam under a uniform load the absolute value of the maximum compressive stress due to bending is, in dimensional form:

$$N_X = \frac{2C_{11}f}{\pi ap} \quad (1 - \text{Cosh}\,(\lambda L))/\text{Cosh}(\lambda L) \tag{3}$$

where $C_{11}$ is the fabric stiffness and $\lambda$ is the characteristic number associated with the differential equation governing this problem and is defined in reference 3. Equation (3) is obtained by evaluating $N_X$ at the beam midpoint. The characteristic number, $\lambda$, in (3) is a function of the pressure, the cross-section radius and the material properties and for the values of these parameters of interest we find that $\lambda L >> 1$. Under this condition the following simplifying assumption can be made with good accuracy

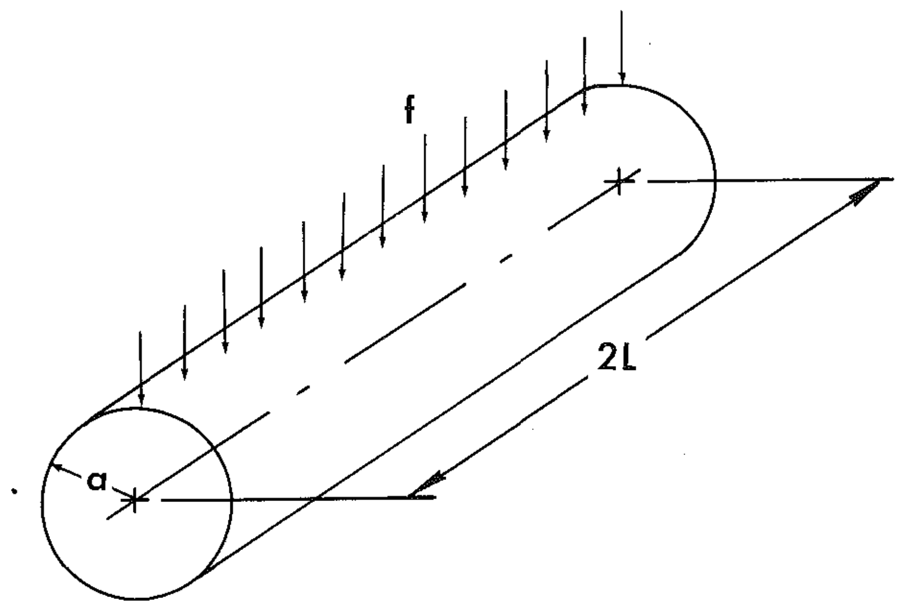
$$\frac{1 - \text{Cosh}\,(\lambda L)}{\text{Cosh}\,(\lambda L)} \cong 1 \tag{4}$$

and the absolute value of the maximum compressive stress is taken to be

$$N_X = \frac{2C_{11}f}{\pi ap} \tag{5}$$

In (5) the parameter f is the intensity of the applied load. The wrinkling load constraint then becomes, by substitution of (5) into (2)

$$pa \geqslant \sqrt{\frac{4f}{\pi}\,C_{11}} \tag{6}$$




d - FABRIC AREA WEIGHT DENSITY

Figure 2. Sketch of beam and nomenclature

To complete the formulation of this constraint we observe that the fabric stiffness, $C_{11}$, is a function of the mass density. That is, the heavier fabrics are stiffer than the lighter ones. This relationship must be put in mathematical form and for lack of more precise information we chose the linear form

$$C_{11} = r_3 d - r_4 \tag{7}$$

with the constants $r_3$ and $r_4$ to be determined from fabric data. Substituting this into (6) the wrinkling load failure constraint becomes

$$pa \geq \sqrt{\frac{4f}{\pi} (r_3 d - r_4)} \tag{8}$$

which is the form to be used in this analysis. The second constraint which assures that the fabric will not rupture under the pressure loading is expressed mathematically as

$$pa \leq N_b \tag{9}$$

where $N_b$ is the fabric breaking strength and pa is the circumferential stress resulting from internal pressure loading. In this analysis we are assuming that the fabric breaking strength is the same in both directions and therefore (9) is the only fabric strength constraint required. This is so because axial stress due to pressurization is one half of the circumferential stress, and if the wrinkling load failure criterion (2) is satisfied, then the maximum tensile axial stress, the sum of that due to pressure and bending, is less than the fabric breaking strength. The fabric breaking strength is also dependent on fabric mass density, and we chose the following linear function to represent this dependence

$$N_b = r_1 d - r_2 \tag{10}$$

and the fabric breaking strength constraint (9) becomes

$$pa \leq r_1 d - r_2 \tag{11}$$

For completely practical reasons we add two constraints, one that keeps the inflation pressure below some maximum and one that keeps the cross-section radius above some minimum. These are written as

$$\begin{aligned} p &\leq p_m \\ a &\geq a_o \end{aligned} \tag{12}$$

The limit on the pressure is thought necessary because of available pressure supplies in the field and the radius limit is necessary because manufacturing techniques will probably have such a limit. This completes the formulation of the problem which we summarize here. Find p, a and d to minimize

$$W = 4\pi a L d \tag{13}$$

subject to the constraints

$$\begin{aligned} pa &\geqslant \frac{4f}{\pi}(r_3 d - r_4) \\ pa &\leqslant r_1 d - r_2 \\ p &\leqslant p_m \\ a &\geqslant a_o \end{aligned} \tag{14}$$

**Solution**

A solution to the problem stated as equations (13) and (14) is the values of p, a, and d which minimizes the function W and satisfy the constraints for specified values of beam length, load, minimum radius, and maximum pressure denoted respectively as L, f, $a_o$ and $p_m$. In addition, the parameters $r_1$, $r_2$, $r_3$, and $r_4$ specifying the relations between the fabric density and its strength and stiffness must be specified. The values used for the $r_i$ in what follows were obtained by fitting curves to experimental data as discussed in the appendix.

To obtain a solution, we begin by noting that the wrinkling load and fabric rupture constraints can be written as functions of two variables, d and pa, if the product pa is treated as a single variable. Considering a beam of length 2L = 5 m and loaded with a force intensity 3000 N/m, we have plotted the two constraint boundaries in Figure 3 as a function of the two variables d and pa. The region of allowable designs in the d, pa space is indicated by the crosshatched area on Figure 3. In this form, however, there is no information about the weight. For a fixed value of pressure, the plot in Figure 3 may be viewed as a plot in the coordinates density and radius. Since the weight is a function of only these two coordinates we can plot lines of constant weight on such a plot. This is done in Figure 4 where plots corresponding to that in Figure 3 for four different pressures are given. On these plots in Figure 4 in addition to the constraints are plots of the curves of constant weight for three values of the weight. Examination of the plots in Figure 4 reveals two facts: As the pressure increases, designs with reduced weight are brought into the design region and the lowest weight design that satisfies the constraint will be at the intersection of the fabric rupture and wrinkling load constraint. Returning now to the problem in the coordinates pa and d we can find this design point

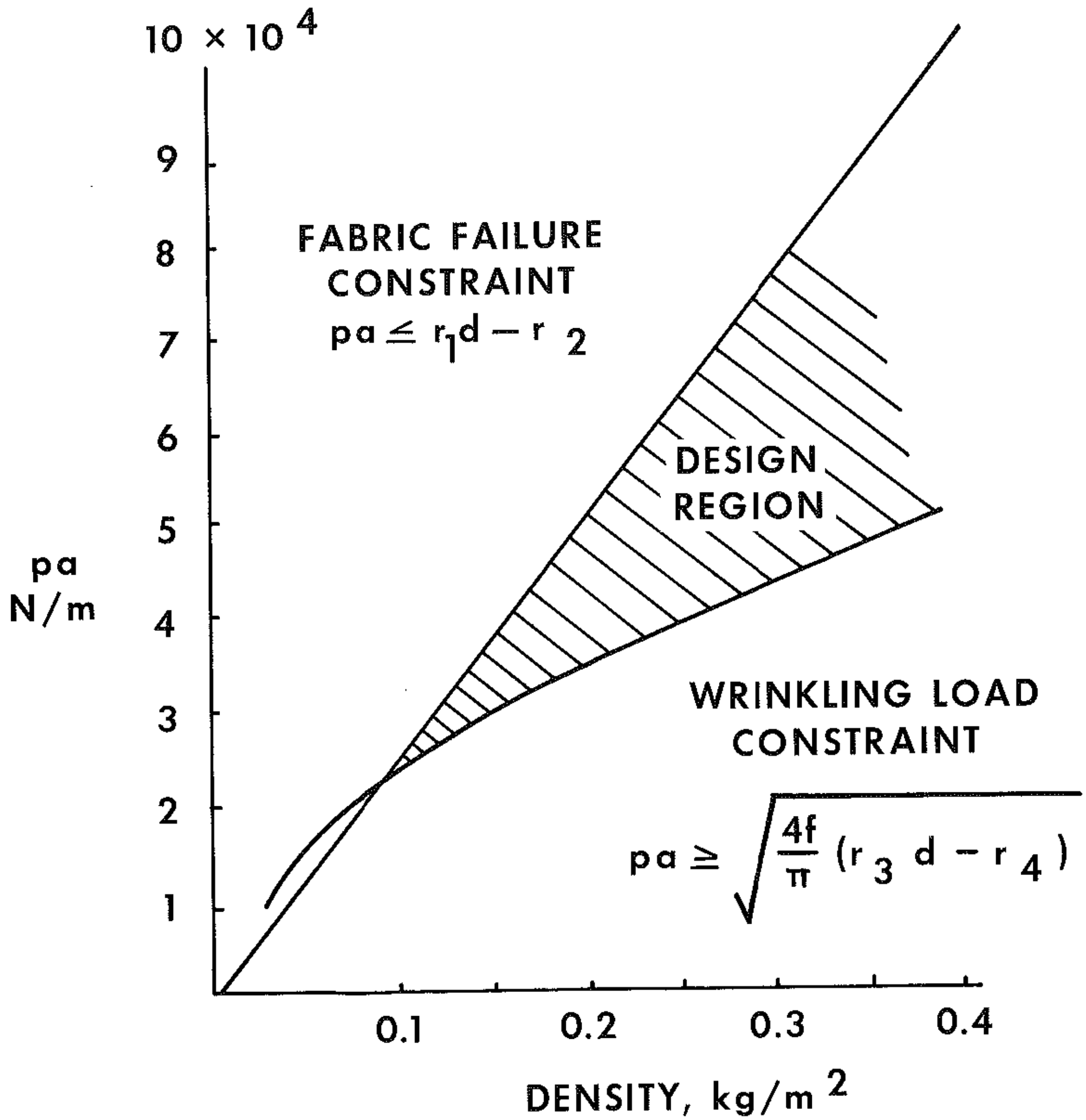



Figure 3. Problem constraints and design region

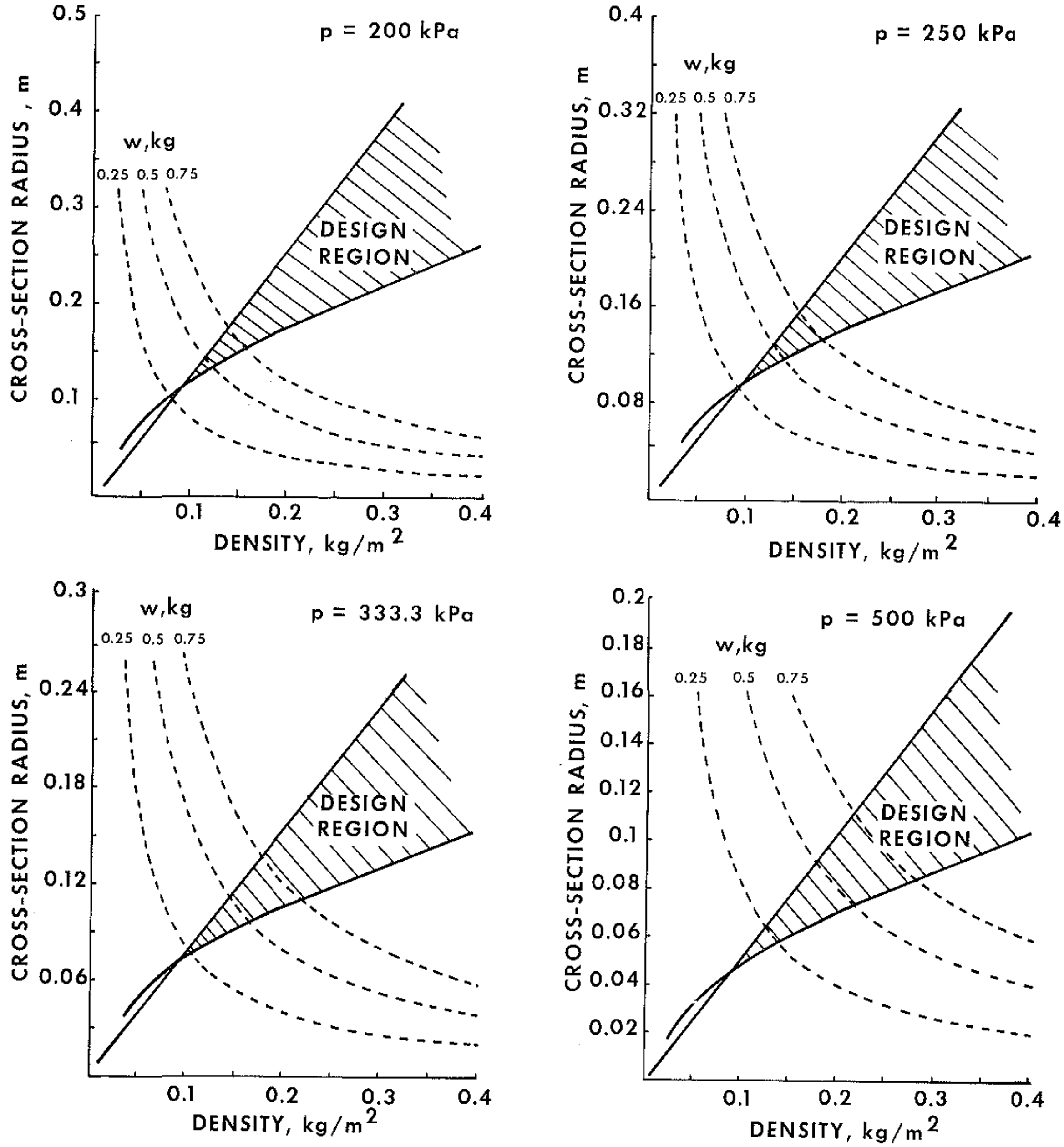



Figure 4. Problem constraints and design region as a function of cross section radius and density for several values of pressure

by solving the equation for the constraint boundaries

$$\begin{aligned} pa &= r_1 d - r_2 \\ pa &= \frac{4f}{\pi} (r_3 d - r_4) \end{aligned} \tag{15}$$

to obtain

$$d = \bar{d} \tag{16}$$

$$pa = A \tag{17}$$

In (16) and (17) $\bar{d}$ and A are the magnitudes of the density and the pressure-radius product of the minimum weight beam. It remains only to find expressions for the pressure and the radius from (17) and the remaining constraints on the radius and pressure. We have thus reduced the problem from that stated in (13) and (14) to the following. Find p and a such that

$$\begin{aligned} pa &= A \\ p &\leqslant p_m \\ a &\geqslant a_o \end{aligned} \tag{18}$$

To obtain the solution of this reduced problem we must consider the two cases shown in Figure 5. In both of these plots the allowable design region is to the right of the line $a = a_o$ and below the line $p = p_m$. Thus in case 1 the solution will be the intersection of the line $a = a_o$ and $pa = A$ giving

$$\begin{aligned} a &= a_o \\ p &= A/a_o \end{aligned} \tag{19}$$

Similarly for case 2, the solution will be the intersection of the line $p = p_m$ and $pa = A$ giving

$$\begin{aligned} p &= p_m \\ a &= A/p_m \end{aligned} \tag{20}$$

This completes the solution with the minimum weight design being given by the solution of (15) and either (19) or (20) to give the values of the cross-section radius, pressure and fabric density corresponding to the minimum weight.

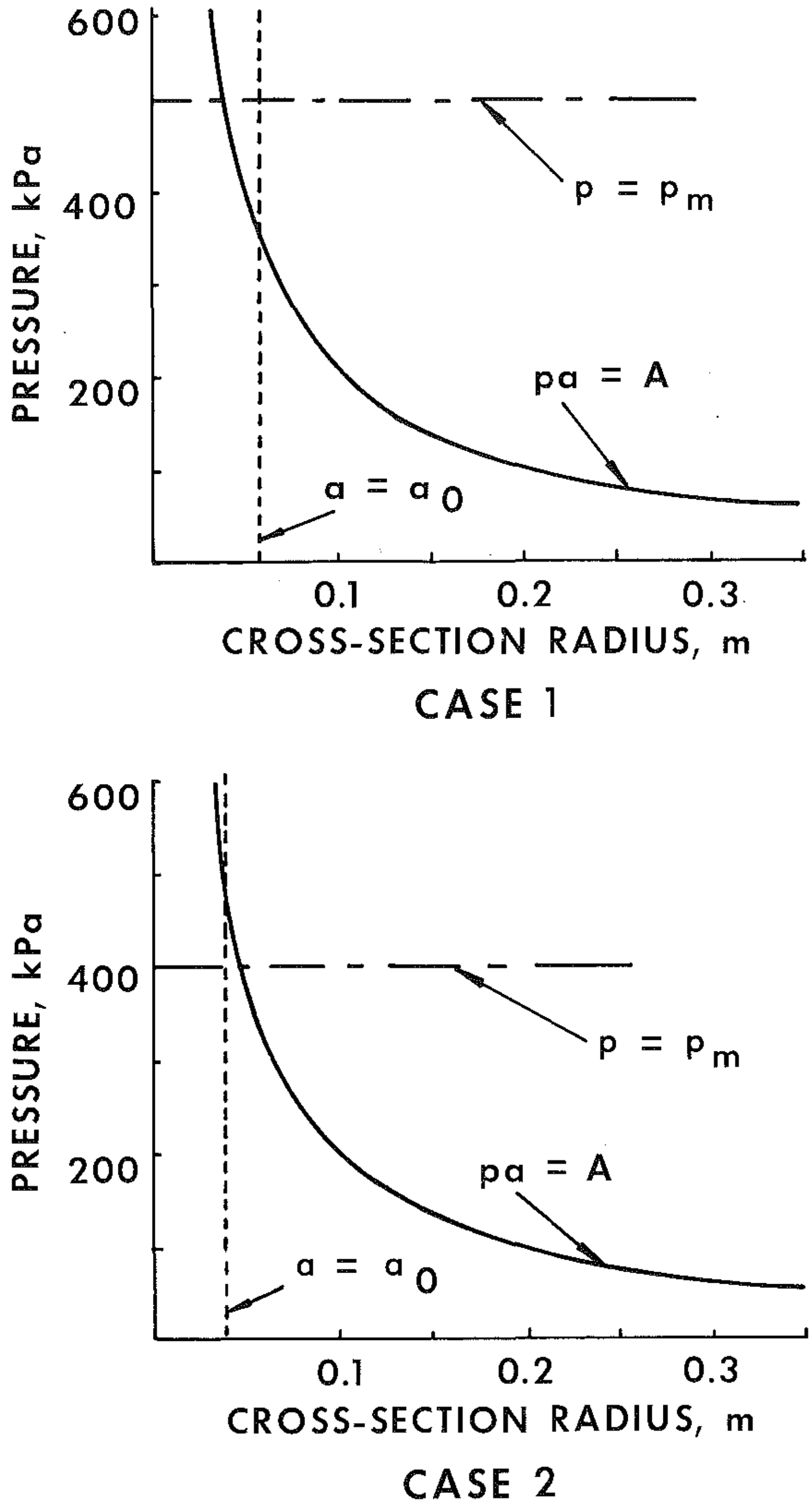



Figure 5. Plots illustrating the solution of the reduced problem

## DISCUSSION

Our purpose here is not to present numerical results giving the minimum weight designs for various parameters but to discuss the general character of the solution. In this regard we will look at the direct effect on minimum weight design of the primary variables, radius, pressure, and fabric density and some inferred results concerning the fabric stiffness and strength characteristics.

Making reference again to Figure 3 and recalling that the design point is at the intersection of the constraint boundaries, it is clear that the minimum weight design corresponds, not surprisingly, to the smallest value of the fabric density within the design region. To see the effect on minimum weight design of the pressure, we examine Figure 4 and find that if the pressure increases, designs having lower weights are brought into the design region. Thus, minimum weight design requires the use of higher pressures, and since at the design point $pa = A$, where A is a constant, the use of higher pressures requires the use of smaller cross-section. However, there is an upper constraint on the pressure and lower constraint on the cross-section radius, and one of these constraints sets the final magnitude of these variables. Thus, we find the minimum weight is obtained by using fabrics of low density and beams having small cross-section radii and large inflation pressures. It should also be noted that the minimum weight design lies at an intersection of the constraint boundaries, a situation very similar to that occurring in linear programing where the solution always occurs at an intersection of the constraints.

Turning now to the effect of fabric strength and stiffness on the minimum weight design, we note on Figure 4 that lower weight is achieved by moving the design point, the intersection of the constraints, to the left. Referring to Figure 3, it can be seen that this can be accomplished by increasing the slope of the fabric failure constraint and decreasing the local slope of the wrinkling load constraint. The slope of the fabric rupture constraint can be increased by increasing the magnitude of the parameter $r_1$, which is the rate of increase of strength with weight. Thus, fabrics which have the highest rates of increase in strength with weight should be favored if minimum weight is desired. Similarly, the wrinkling constraint local slope can be decreased by decreasing the magnitude of the parameter $r_3$ which is the rate of change of stiffness with weight. Thus, for minimum weight design, fabrics having lower stiffness are favored.

One small difficulty with the analysis should be pointed out, and this concerns the wrinkling load constraint. It will be noticed that in this constraint an imaginary solution results when $r_3 d < r_4$ and such a result makes no sense. The difficulty is not fundamental with the wrinkling load criterion but rather with the modelling of the fabric stiffness as a function of fabric weight for very small weights. For the data given in the appendix, this difficulty occurs at a density of 0.0015 $kg/m^2$ which is way below the range of interest. Thus, although the difficulty is theoretically present, it is of no real concern in actual physical problems. This all points out the need for a more comprehensive data base for the strength and stiffness of fabrics which can be used to generate a more realistic and accurate model of these parameters as a function of weight.

## CONCLUSIONS

A study of the minimum weight design of pressure-stabilized beams has been completed. The minimization was done subject to four inequality constraints and yielded the inflation pressure, cross-section radius, and fabric density corresponding to the minimum weight design. The solution of this problem reveals that minimum weight is obtained with large values of inflation pressure, small values of cross-section radius and, not surprisingly, low values of fabric density. From this solution it is shown that fabrics of high strength and low stiffness per unit density favor minimum weight.

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## APPENDIX

### Determination of Constants Representing the Fabric Strength and Stiffness as a Function of its Density

In the body of the report we used the following linear representations of the fabric strength, $N_b$, and stiffness, $C_{11}$, in terms of its area mass density, d.

$$N_b = r_1 d - r_2$$

$$C_{11} = r_3 d - r_4$$

It is our purpose here to obtain magnitudes for the parameters $r_i$, i = 1 through 4. This is accomplished by fitting curves to experimental data from three nylon fabrics, two very light fabrics and one heavy fabric. These data are given in the table followed by the parameter values obtained by a least squares fit of the data.

| Density ($kg/m^2$) | Breaking Strength N/m | Stiffness N/m |
|---|---|---|
| 0.041 | $8.76 \times 10^3$ | $53.8 \times 10^3$ |
| 0.041 | 4.55 | 24.9 |
| 0.034 | 8.06 | 53.8 |
| 0.034 | 8.06 | 41.2 |
| 0.494 | 126.00 | 737.0 |
| 0.494 | 131.00 | 933.0 |

$r_1 = 2.65 \times 10^5$ $\qquad r_3 = 17.4 \times 10^5$

$r_2 = 2.58 \times 10^3$ $\qquad r_4 = 27.0 \times 10^3$

## SYMBOLS

| | |
|---|---|
| $A$ | a constant defined by equation (17) |
| $a$ | cross-section radius |
| $a_o$ | minimum allowable radius |
| $C_{11}$ | fabric stiffness |
| $d$ | fabric mass density |
| $f$ | applied load intensity |
| $L$ | beam half length |
| $N_b$ | fabric breaking strength |
| $N_x$ | axial stress resultant |
| $p$ | inflation pressure |
| $p_m$ | maximum allowable inflation pressure |
| $r_1$, $r_2$ | parameters relating fabric strength and density |
| $r_3$, $r_4$ | parameters relating fabric stiffness and density |
| $W$ | beam weight |
| $\lambda$ | characteristic number |